

## L6 Assessment 2

① a) i)  $\frac{1}{16} = 2^a$

$$\frac{1}{2^4} = 2^{-4} \quad \therefore a = -4$$

ii)  $3\sqrt[3]{64} = 2^b$

$$3\sqrt[3]{2^6} = 2^2 \quad \therefore b = 2$$

b)  $3\sqrt[3]{64} \times 2^x = \frac{1}{16}$

$$2^2 \times 2^x = 2^{-4}$$

$$2^{2+x} = 2^{-4} \Rightarrow 2+x = -4$$

$$x = -6$$

②

$$y = 3x^3 - 2x^2$$

$$\frac{dy}{dx} = 9x^2 - 4x$$

$$x = -1$$

$$y = 3(-1)^3 - 2(-1)^2 = -3 - 2 = -5$$

$$m_T = \left. \frac{dy}{dx} \right|_{x=-1}$$

$$= 9(-1)^2 - 4(-1) = 9 + 4 = 13$$

$$\therefore m_N = -\frac{1}{13}$$

$$m = -\frac{1}{13} \quad (-1, -5) \Rightarrow y+5 = -\frac{1}{13}(x+1)$$

$$13y + 65 = -x - 1$$

$$\therefore x + 13y + 66 = 0$$

③

$$L_1: 4y - 3x = 10$$

$$4y = 3x + 10$$

$$y = \frac{3}{4}x + \frac{10}{4} \quad m_1 = \frac{3}{4}$$

$$L_2: m_2 = \frac{8+1}{-1-3} = \frac{9}{-6} = -\frac{3}{2}$$

Neither as  $m_1 \neq m_2$  so not parallel.  
 $m_1 \times m_2 \neq -1$  so not perpendicular.

(4)

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

a) i)  $(x-3)^2 - 9 + (y+5)^2 - 25 + 9 = 0$

Centre is  $(3, -5)$

ii)  $(x-3)^2 + (y+5)^2 = 25 \quad \therefore r = 5$

b)

$$y = kx$$

$$x^2 + (kx)^2 - 6x + 10kx + 9 = 0$$

$$x^2 + k^2 x^2 - 6x + 10kx + 9 = 0$$

Two distinct points of intersection  $\Rightarrow$  two distinct solutions  $\Rightarrow b^2 - 4ac > 0$

$$a = 1 + k^2$$

$$(10k-6)^2 - 4(1+k^2)9 > 0$$

$$b = 10k-6$$

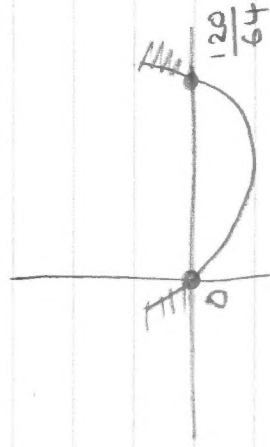
$$100k^2 - 120k + 36 - 36(1+k^2) > 0$$

$$c = 9$$

$$100k^2 - 120k + 36 - 36 - 36k^2 > 0$$

$$64k^2 - 120k > 0$$

$$k(64k - 120) > 0$$



{or use the  
calculator!}

$$k < 0 \text{ or } k > \frac{15}{8}$$

(5)

$$f(x) = x^3$$

$$f(x+h) = (x+h)^3$$

$$f'(x) = \lim_{h \rightarrow 0} \left[ \frac{(x+h)^3 - x^3}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{3x^2h + 3xh^2 + h^3}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{h(3x^2 + 3xh + h^2)}{h} \right]$$

$$= \lim_{h \rightarrow 0} [3x^2 + 3xh + h^2] = \underline{\underline{3x^2}}$$

(6) a)  $L_1: kx + (1-k)y = 5$

$$(1-k)y = 5 - kx$$

$$y = \frac{5 - kx}{1-k} = -\frac{kx}{1-k} + \frac{5}{1-k}$$

$$\therefore m_1 = -\frac{k}{1-k}$$

b)  $L_2 (1, 4) (3, k)$

$$m_2 = \frac{k-4}{2}$$

c) i)  $m_1 \times m_2 = -1 \Rightarrow -\frac{k}{1-k} \times \frac{k-4}{2} = -1$

$$-\frac{k(k-4)}{2(1-k)} = -1$$

$\Rightarrow$

$$\frac{-k^2 + 4k}{2-2k} = -1$$

$$-k^2 + 4k = -2 + 2k$$

$$\Rightarrow k^2 - 2k - 2 = 0$$

c) ii)  $k = 1 \pm \sqrt{3}$

⑦ C(-2,6) (10,11)

a)  $(x+2)^2 + (y-6)^2 = r^2$

Sub in (10,11)

$$(10+2)^2 + (11-6)^2 = r^2$$

$$144 + 25 = 169 \quad \therefore r^2 = 169$$

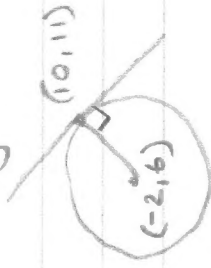
$$(x+2)^2 + (y-6)^2 = 169$$

Sub in (10,1)

$$(10+2)^2 + (1-6)^2$$

$$= 144 + 25 = 169 \quad \therefore \text{the circle also passes through (10,1).}$$

b) Tangent at (10,11):



$$m_r = \frac{11-6}{10-(-2)} = \frac{5}{12}$$

$$\therefore m_T = -\frac{12}{5}$$

$$y - 11 = -\frac{12}{5}(x - 10)$$

$$5y - 55 = -12x + 120$$

$$12x + 5y = 175$$



Tangent at  $(0,1)$ :

$$m_r = \frac{1-6}{10--2} = -\frac{5}{12}$$

$$\therefore m_T = \frac{12}{5}$$

$$y-1 = \frac{12}{5}(x-10)$$

$$5y-5 = 12x-120$$

$$115 = 12x-5y$$

meets y-axis when  $x=0$

$$T_1: 12x+5y=115$$

$$5y=115$$

$$y=23$$

$$T_2: 12x-5y=115$$

$$-5y=115$$

$$y=-23$$

So P is  $(0, 23)$

Q is  $(0, -23)$

Hence PQ is  $23+23=46$

(8)

$$y = 2x^3 + 6x^2 - 12x + 3$$

$$\frac{dy}{dx} = 6x^2 + 12x - 12$$

$$y = 1 + 60x - 6x^2$$

$$\frac{dy}{dx} = 60 - 12x$$

Chris says  $6x^2 + 12x - 12 > 60 - 12x$

His claim is not true when  $6x^2 + 12x - 12 \leq 60 - 12x$

$$6x^2 + 24x - 72 \leq 0$$

$$-6 \leq x \leq 2$$

Hence Chris is wrong when  $-6 \leq x \leq 2$ .

